ДИС Кирчев 6.12.12

cos x sin x
$f(x) = (1+x)^\alpha$ (при $\alpha = n$ - бином на Нютон)

$f(x) = (1+x)^\alpha = e^{\alpha \ln(1+x)}$ (трябва $\ln(1+x) > 0$ задължително)

$f'(x) = \alpha(1+x)^{\alpha-1}$
$f''(x) = \alpha(\alpha-1)(1+x)^{\alpha-2}$
$\vdots$
$f^{(n)}(x) = \alpha(\alpha-1)\dots(\alpha-n+1)(1+x)^{\alpha-n}$
$f^{(n+1)}(\theta x) = \alpha(\alpha-1)\dots(\alpha-n)(1+\theta x)^{\alpha-n-1}$

Остатъчен член R_n по Коши, защото при Лагранж не работи при $x < 1$

$1+x > 0$
$-1 < x < 1$ т.е. $x \in (-1, 1)$

$$R_n = \frac{(x-a)^{n+1}}{n!} f^{(n+1)}(a+\theta(x-a))(a-\theta)^n$$

$$R_n = \frac{x^{n+1}}{n!} f^{(n+1)}(\theta x)(1-\theta)^n$$

$$\Rightarrow \frac{x^{n+1}(1-\theta)^n}{n!} \alpha(\alpha-1)\dots(\alpha-n)(1+\theta x)^{\alpha-n-1} =$$

$$\underbrace{\frac{x^{n+1}}{n!}\alpha(\alpha-1)\dots(\alpha-n)}_{a_n^1} \cdot \underbrace{\left(\frac{1-\theta}{1+\theta x}\right)^n}_{a_n^2} \cdot \underbrace{(1+\theta x)^{\alpha-n-1}}_{a_n^3}$$

Разглеждаме по-подробно

Разглеждаме a_n^2 $\qquad \frac{1-\theta}{1+\theta x} < 1$

$$\left.\begin{array}{l} -1 < x \quad , \; 0 < x+1 \\ 0 < \theta x + \theta \; , \; -\theta < \theta x \\ 1-\theta < 1+\theta x \end{array}\right\} \Rightarrow \quad < 1$$

Разглеждаме a_n^3

$$a_n^3 \leq (1, 1+\theta x)^{\alpha-1} < C$$

Разглеждаме a_n^1

$$\frac{*1}{1+\theta x} < \frac{1}{1-\theta} \quad \text{от} \quad \frac{1-\theta}{1+\theta x} < 1 \quad \Rightarrow$$

$$\frac{|x|}{(1+\theta x|} < \frac{|x|}{1-\theta}$$

Разглеждаме

$$\left|\frac{\alpha(\alpha-1)\ldots(\alpha-n)x^{n+1}}{n!}\right| \underset{\text{следва от съкращение}}{=} \frac{|\alpha-n-1|}{n+1}\,|x| =$$

$$= \frac{n\left|\frac{\alpha}{n_0} - 1 - \frac{1}{n_0}\right|}{n+1}\,|x| \longrightarrow |x| \quad \text{от}$$

$$\frac{n}{n+1} \to 1 \; ; \; \frac{\alpha}{n_0} \to 0 \; ; \; \frac{1}{n_0} \to 0$$

$$(1+x)^\alpha = \sum_{n=1}^{\infty} \frac{\alpha(\alpha-1)\cdots(\alpha-n)}{n!} x^n \Rightarrow \sum_{n=1}^{\infty} \binom{\alpha}{n} x^n$$

биномен ред

$$f^{(n)}(0) = \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}$$

$$f^{(n)}(x) = \left((1+x)^\alpha\right)^{(n)} = \alpha(\alpha-1)\cdots(\alpha-n+1)(1+x)^{\alpha-n}$$

$$f(x) = \ln(1+x)$$

$$f'(x) = \frac{1}{1+x} = (1+x)^{-1} \Rightarrow \alpha = -1$$

$$f^{(n)}(x) = \alpha(\alpha-1)\cdots(\alpha-(n-1)+1)(1+x)^{\alpha-n+1} =$$

$$f^{(n)}(x) = \alpha(\alpha-1)\cdots(\alpha-n+2)(1+x)^{\alpha-n+1} \Rightarrow$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \binom{\alpha}{n} x^{n+1} = \sum_{n=1}^{\infty} \binom{-1}{n} x^{n+1}$$

от формулата на Тейлор

$$f(0) = \ln 1 = 0 \qquad \frac{f^{(n)}(0)\, x^{n+1}}{n+1}$$

$$\binom{-1}{1} = -1 \quad , \quad \binom{-1}{2} = \frac{(-1)(-2)}{2} = 1$$

$$\binom{-1}{3} = \frac{(-1)(-2)(-3)}{3.2} = -1$$

$$\sum_{n=0}^{\infty} \frac{n!\, x^n (-1)^n}{(n+1)!} = \frac{x^n(-1)^{n-1}}{n} = \ln(1+x)$$

$x - \frac{x^2}{2} - \frac{x^3}{3} - \dots$

$-1 < x < 1$ няма при $x \notin (-1,1)$

няма решение

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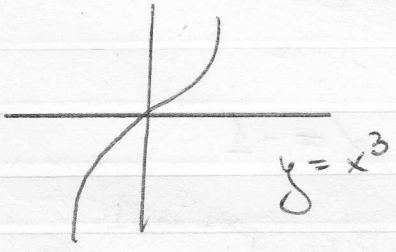
x_0

$x \in (x_0 - \varepsilon, x_0 + \varepsilon)$

$f(x) \leq f(x_0)$ max

$f(x) \geq f(x_0)$ min

Ферма



$f'(x_0) = 0$

$\frac{f(x_0+h) - f(x_0)}{h} \to f(x_0) \Rightarrow f(x_0+h) > f(x_0)$

$f(x_0+h) - f(x_0) > 0$

$(- \; | \; +)$ min $\quad (+ \; | \; -)$ max

Ако $f''(x_0) > 0$ min то $f'(x_0+h) - f'(x_0) > 0 \Rightarrow$

или $f'(x_0+h) - f(x_0) < 0 \Rightarrow$ $\boxed{f'(x_0+h) \geq f(x_0)}$

$f'(x_0+h) - f'(x) = 0 \Rightarrow h < 0$